A Study on the Determination of Optimal Reference Spectrum for Random Vibration Control in Environmental Vibration Test

Jun-Yeop Kim,* Eul-Jae Yoon,* Sung-Jo Chang,* Do-Young Kim* and Weui-Bong Jeong**

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For random vibration control in environmental vibration test it is desired that the spectra at the specimen attachment points coincide with the specified reference spectrum. However, this goal is seldom achieved because of the dynamic characteristics of shaker, fixture, and specimen and of the interactions between them in the test frequency ranges. This paper proposes a method for the determination of the optimal reference spectrum which minimizes the spectral deviations between the specified reference spectrum and spectra at the specimen attachment points. The least square method is used for this purpose. This method requires only the measured FRFs at the control point and specimen attachment points in pre-vibration test and is shown through theoretical and experimental results to be effective.

Key Words : Environmental Vibration Test, Random Vibration Control, Reference Spectrum, Fixture, Mechanical Impedance

1. Introduction

Environmental vibration test becomes necessary in order to determine the resistance of equipment to the deterious effects of natural and induced environments, and to find out weak components which could generate failures, malfunctions or improper operations during service. Currently military and other test specifications call for random vibration test of structures, because many structures behave much differently when vibrating in a random manner than they do with sinusoidal motion.

For random vibration control in environmental vibration test it is desired that the spectra at the specimen attachment points coincide with the specified reference spectrum. However, this goal is seldom achieved because of the dynamic characteristics of shaker, fixture and specimen and of the interactions between them in the test frequency range. (Tomlinson, 1979; Rao, 1987; Tomlinson, 1987; Ewins, 1984; Olson, 1986; Brown and Allemang, 1989) Therefore, PSD (Power Spectral Density) at the control point is compared with the specified reference spectrum and a correction or modification of the driving spectrum is generated, so that a closed loop is utilized in random vibration control system. (Harris and Crede, 1976) However, this method controls only the control point in accordance with the specified reference spectrum, not the specimen attachment points, so that the specimen undergoes the undertest and/or overtest.

Sweitzer(1987) proposed a technique to correct for mechanical impedance effects during vibration test and presented experimental results for a flight justification test using corrected random input spectrum. He treated single degree of freedom specimen installed on rigid fixture over the test frequency range 10Hz-1kHz. If fixture as well as specimen appears rigid body motion, mechanical

*Agency for Defense Development, Taejeon, Korea
**Department of Mechanical and Precision Engineering and Research Institute of Mechanical Technology, Pusan University, Pusan, Korea

impedance will be the same in all points regardless of the control points and specimen attachment points. Therefore, it is possible to control that the spectra at the specimen attachment points coincide with the specified reference spectrum. But mechanical impedance differs at every point where control accelerometer is mounted, because fixture as well as specimen generally shows the behavior of multi degree of freedom system. Therefore, control technique which considers simulateneously the control point and specimen attachment points is required.

This paper proposes a method for the determination of optimal reference spectrum at the control point, which regards the control point and specimen attachment points in random vibration control. The least square method is used to minimize the spectral deviations between the specified reference spectrum and spectra at the specimen attachment points. This method requires only the measured FRFs(Frequency Response Functions), which are the ratio of PSD at the control point and specimen attachment points to that at the armature table. And the proposed method is proved to be effective through experimental verification by comparing with the results of the existing one.

2. Theoretical Background

2.1 FRF in random vibration test

PSD, *in* G^2/Hz, is used as a reference spectrum in random vibration control of environmental vibration test. Figure 1 shows a functional block diagram of a digitally controlled random vibration system.

The response spectrum vector relating the motion of the structure to random signal is given by

$$S_{x_l}(\omega_k)=\sum_{j=1}^{N_f}[\,|H_{lj}(\omega_k)|^2\cdot S_{f_j}(\omega_k)]\quad \text{for } l=1,\ \cdots,\ N_r \tag{1}$$

where,

$l=1, 2,\cdots, N_r$: *number of response points*
$j=1, 2,\cdots, N_f$: *number of excitation forces*
$k=1, 2,\cdots, N_\omega$: *number of discrete frequencies*
ω : *angular frequency* (rad/s)
$S_{x_l}(\omega_k)$: *response displacement spectrum at* $\omega=\omega_k$
$S_{f_j}(\omega_k)$: *excitation force spectrum at* $\omega=\omega_k$
$H_{lj}(\omega_k)$: *system FRF at* $\omega=\omega_k$

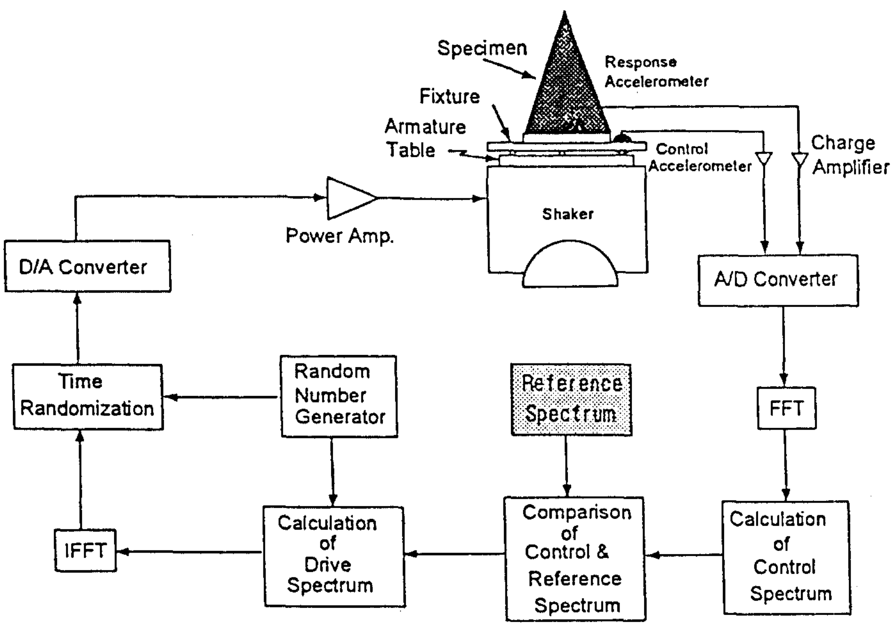


Fig. 1 Block diagram of random vibration control system

Assuming that the diaphragming mode (Klee, et. al., 1971) (or oil canning mode) of the armature table is not occurred over the test frequency range so that the armature table experiences rigid body motion, then the system is considered to be single input system to which same exciting forces are transmitted through the bolts connecting the armature table and fixture. In addition, as shown in Fig. 2 the control acceleration spectrum at the control point $S_a(\omega_k)$ is measured instead of excitation force spectrum $S_{f_j}(\omega_k)$ as an input and response acceleration spectrum $S_{\ddot{x}l}(\omega_k)$ instead of response displacement spectrum $S_{x_l}(\omega_k)$ as an output of environmental vibration test.

Taking into account of those, Eq. (1) can be written by the followings :

$$S_{x_l}(\omega_k) = |H_{la}(\omega_k)|_l^2 \cdot S_a(\omega_k) \quad \text{for } l=1, \cdots, N_r \tag{2}$$

where, $H_{la}(\omega_k)$ is FRF, i.e. the ratio of PSD at arbitary point on the fixture to that at the armature table. Separating $S_{\ddot{x}l}(\omega_k)$ of Eq. (2) into the spectrum at one control point $S_c(\omega_k)$ and the spectra at the specimen attachment points $S_{s_i}(\omega_k)$, Eq. (2) becomes

$$\left\{ \begin{array}{c} S_c(\omega_k) \\ \hdashline S_{s_i}(\omega_k) \end{array} \right\} = \left\{ \begin{array}{c} |H_c(\omega_k)|^2 \\ \hdashline |H_{s_i}(\omega_k)|^2 \end{array} \right\} \cdot S_a(\omega_k) \tag{3}$$

where,

$H_c(\omega_k)$: *FRF at the control point at* $\omega=\omega_k$ *(the ratio of PSD at the control point on the fixture to that at the armature table)*

$H_{s_i}(\omega_k)$: *FRFs at the specimen attachment points at* $\omega=\omega_k$ *(the ratio of PSD at the specimen attachment points on the fixture to that at the armature table)*

$i=1, 2, \cdots, N_s$: *number of specimen attachment points*

2.2 Derivation of optimal reference spectrum

The spectra at the specimen attachment points are much different from the specified reference spectrum because of the dynamic characteristics and interactions of shaker, fixture, and specimen in random vibration control. It is then required to minimize the spectral deviations between the specified reference spectrum and spectra at the specimen attachment points, and this can be expressed by means of least square method as follows :

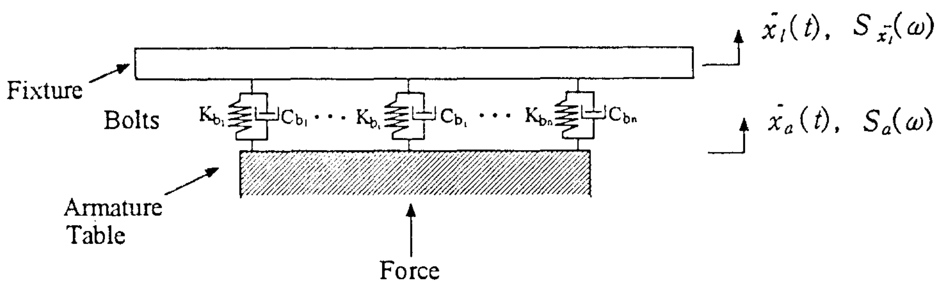
$$\text{minimize } J = \sum_{i=1}^{N_s} (S_{s_i}(\omega_k) - S_{ref}(\omega_k))^2 \tag{4}$$

where,

J : *object function*

$S_{s_i}(\omega_k)$: *response acceleration spectra at the specimen attachment points* $[G^2/Hz]$

$S_{ref}(\omega_k)$: *specified reference spectrum* $[G^2/Hz]$



$\ddot{x}_a(t)$: *Acceleration at Armature Table*
$\ddot{x}_l(t)$: *Acceleration at Fixture*
$S_a(\omega)$: *Control Acceleration Spectrum at Armature Table*
$S_{\ddot{x}l}(\omega)$: *Response Acceleration Spectrum at Fixture*
K_{b_i} : *i-th Equivalent Spring Constant*
C_{b_i} : *i-th Equivalent Damping Coefficient*

Fig. 2 Vibration test model

Equation (4) can be also written in the following form by using Eq. (3) :

$$minimize\ J=\sum_{i=1}^{N_S}(|H_{s_i}(\omega_k)|^2\cdot S_a(\omega_k)-S_{ref}(\omega_k))^2 \tag{5}$$

To obtain the minimum value for J, which is a function of $S_a(\omega_k)$, we set the following partial derivatives to zero :

$$\frac{\partial J}{\partial S_a(\omega_k)}=0 \tag{6}$$

This yields optimal acceleration spectrum at the armature table. If we represent this optimal acceleration spectrum as $S_{ma}(\omega_k)$ in order to distinguish from $S_a(\omega_k)$, $S_{ma}(\omega_k)$ is given as follows :

$$S_{ma}(\omega_k)=\frac{S_{ref}(\omega_k)\cdot\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^2}{\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^4} \tag{7}$$

The optimal reference spectrum at the control point is also obtained using FRF at the control point, $|H_c(\omega_k)|$ and $S_{ma}(\omega_k)$, and expressed as follows :

$$S_{mc}(\omega_k)=|H_c(\omega_k)|^2\cdot S_{ma}(\omega_k)$$

Therefore, by substituting $S_{ma}(\omega_k)$ optimal reference spectrum $S_{mc}(\omega_k)$ can be written as follows :

$$S_{mc}(\omega_k)=\frac{S_{ref}(\omega_k)\cdot|H_c(\omega_k)|^2\cdot\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^2}{\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^4} \tag{8}$$

2.3 Calculation of response acceleration spectra

2.3.1 Spectra by the existing method

Random vibration control is conducted such that PSD at the control point $S_c(\omega_k)$ coincides with the specified reference spectrum $S_{ref}(\omega_k)$ in the existing method-i. e.,

$$S_c(\omega_k)=S_{ref}(\omega_k)$$

In this case, the acceleration spectrum at the armature table $S_a(\omega_k)$ can be expressed from Eq. (3) as follows :

$$S_a(\omega_k)=\frac{S_c(\omega_k)}{|H_c(\omega_k)|^2}=\frac{S_{ref}(\omega_k)}{|H_c(\omega_k)|^2} \tag{9}$$

As a result, the response acceleration spectra at the specimen attachment points can be expressed using Eqs. (3) and (9), so that

$$S_{s_i}(\omega_k)=|H_{s_i}(\omega_k)|^2\cdot S_a(\omega_k)=\frac{|H_{s_i}(\omega_k)|^2}{|H_c(\omega_k)|^2}\cdot S_{ref}(\omega_k) \tag{10}$$

2.3.2 Spectra by the proposed method

When controlled using optimal reference spectrum as reference spectrum, the response acceleration spectra at the specimen attachment points can be expressed from Eqs. (3) and (7) as follows :

$$S_{ms_i}(\omega_k)=|H_{s_i}(\omega_k)|^2\cdot S_{ma}(\omega_k)=\frac{S_{ref}(\omega_k)\cdot|H_{s_i}(\omega_k)|^2\cdot\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^2}{\sum_{i=1}^{N_S}|H_{s_i}(\omega_k)|^4} \tag{11}$$

3. Application and Vibration Test Results

3.1 Measurement of FRFs

To begin with, FRFs at the control point and specimen attachment points have to be measured so that the optimal reference spectrum at the control point has to be determined.

The fixture used to measure FRFs, on which a corn type specimen is mounted as in Fig. 1, is shown in Fig. 3. One point on the fixture was specified as the control point (denoted by "C_1") and three points as the specimen attachment points (denoted by "S_1", "S_2", "S_3"). One control accelerometer is mounted on the armature table, and the fixture is bolted with 17 bolts (3/8″-16 UNC, Length 2″) and with recommended torque value of 280 in-lbf. After that, FRFs at the control point and specimen attachment points with respect to the armature table of 0. 001 G^2/Hz are measured over the frequency range 10Hz to 2kHz. Measurement system is shown in Fig. 4, and in order to measure FRFs, accelerations are picked-up by accelerometers whose signals were analyzed by HP3565S Signal Processing System and

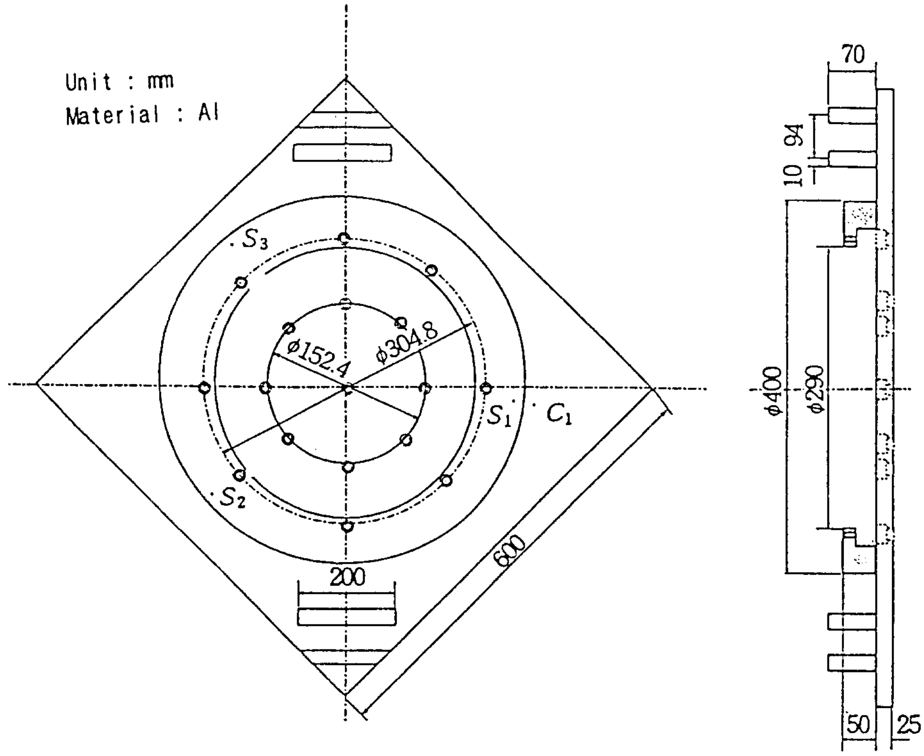


Fig. 3 Dimensions of fixture

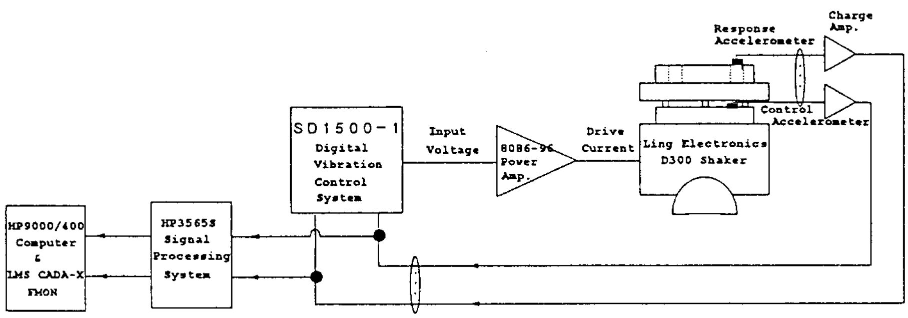


Fig. 4 Block diagram of vibration test

LMS CADA-X FMON software. To reduce errors induced from measurement, we computed FRFs with 30 averages and the frequency resolution was 5 Hz. FRFs at the control point and specimen attachment points are shown in Fig. 5.

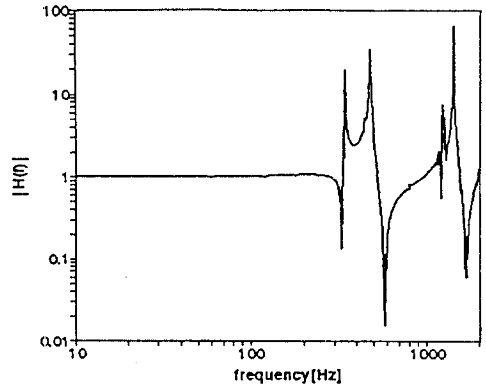
3.2 Determination of optimal reference spectrum

Using the measured FRFs and specified reference spectrum of $0.01 G^2/Hz$, the acceleration spectrum at the armature table $S_{ma}(\omega_k)$, which is a quantity proportional to the excitation force (or driving current of shaker), is determined and is shown in Fig. 6.

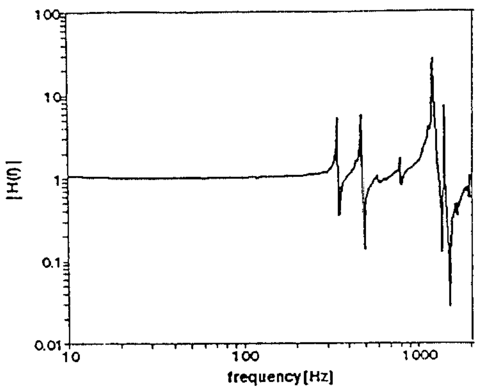
Optimal reference spectrum can be obtained from Eq. (8) and the comparison of optimal reference spectrum and specified reference spectrum is shown in Fig. 7.

3.3 Simulations and experimental results

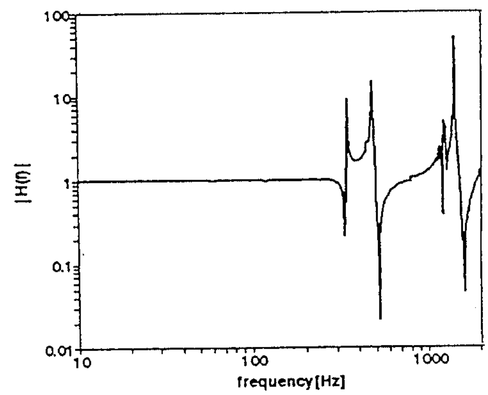
The response acceleration spectra at the



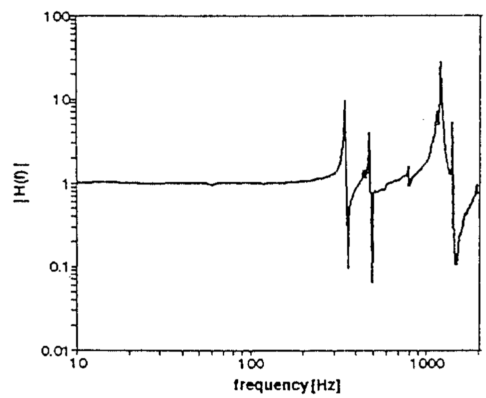
(a) FRF at control point (denoted by "C_1" in Fig. 3)



(c) FRF at specimen attachment point #2 (denoted by "S_2" in Fig. 3)



(b) FRF at specimen attachment point #1 (denoted by "S_1" in Fig. 3)



(d) FRF at specimen attachment point #3 (denoted by "S_3" in Fig. 3)

Fig. 5 Measured frequency response functions

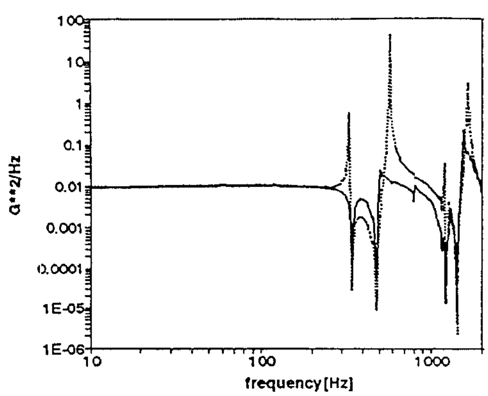


Fig. 6 PSD at armature table in controlled by the existing and proposed method (Dot : existing, Solid : proposed)

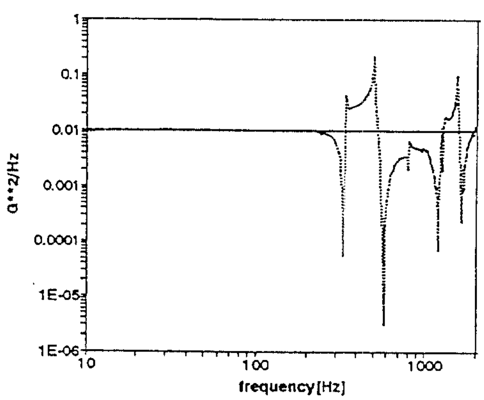
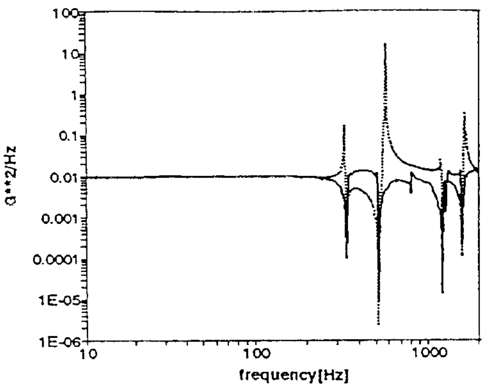


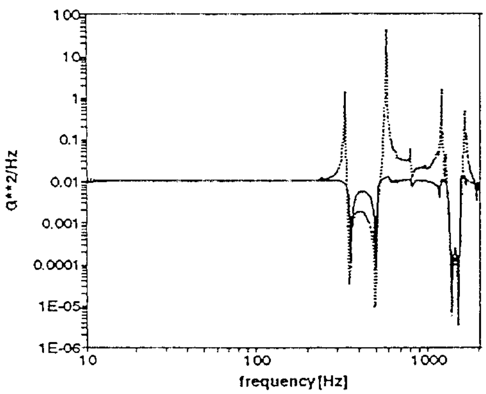
Fig. 7 Comparison of reference spectrum of the existing and proposed method (Solid : existing, Dot : proposed)

specimen attachment points in controlled by the existing and proposed method can be predicted from Eqs. (10) and (11) respectively. Figure 8 shows the comparison of each cases.

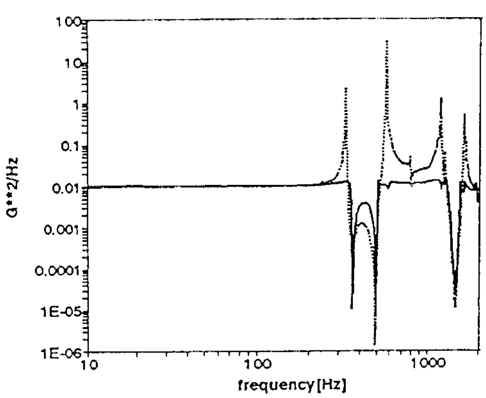
As shown in Fig. 8, when controlled by the specified reference spectrum, specimen undergoes the undertest and/or overtest. However,



(a) Comparison of controlled PSD at specimen attachment point #1, S_1

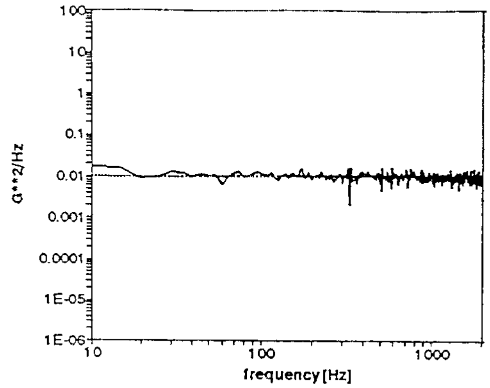


(b) Comparison of controlled PSD at specimen attachment point #2, S_2

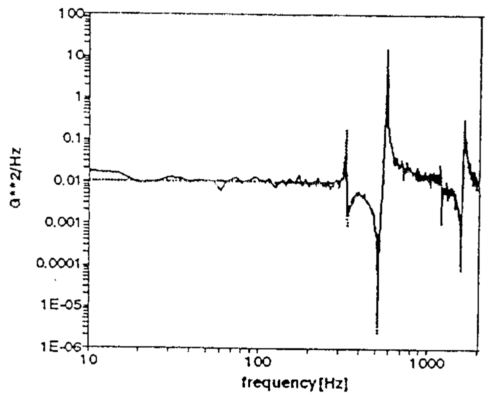


(c) Comparison of controlled PSD at specimen attachment point #3, S_3

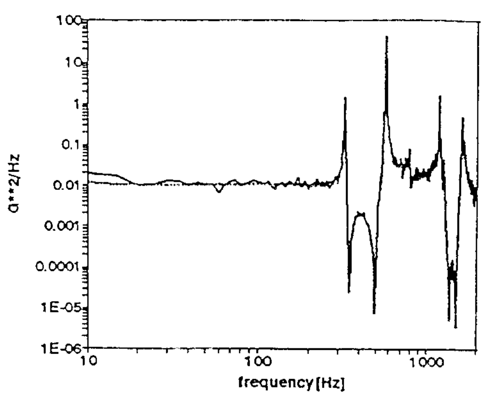
Fig. 8 Computer Simulation Results(Dot : existing, Solid : proposed)



(a) Controlled PSD at control point, C_1

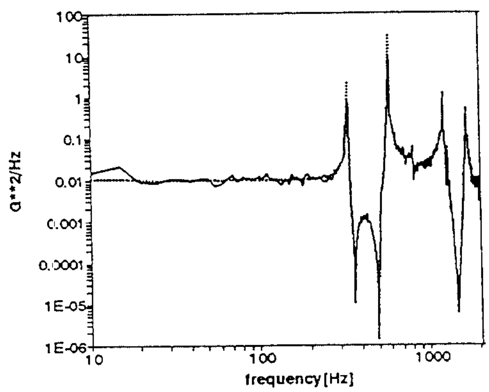


(b) Controlled PSD at specimen attachment point # 1, S_1



(c) Controlled PSD at specimen attachment point # 2, S_2

Fig. 9 Continued



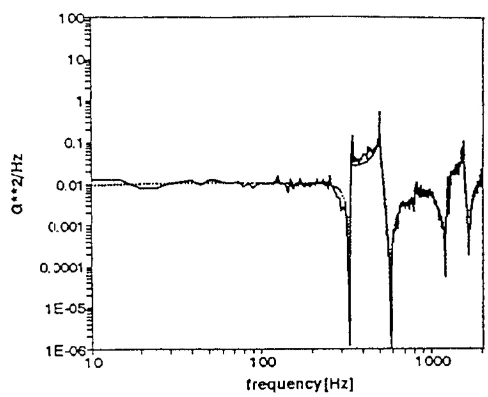
(d) Controlled PSD at specimen attachment point # 3, S_3

Fig. 9 Comparison of theoretical and experimental results of the existing method (Dot : theoretical, Solid : experimental)

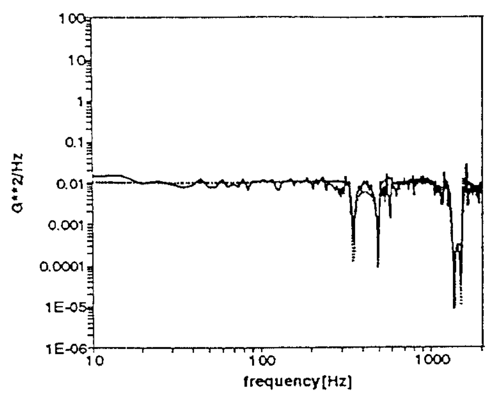
when controlled by optimal reference spectrum proposed, specimen shows the closest motion to the specified reference spectrum over the test frequency range, except for near antiresonance points.

Random vibration control was conducted to prove the validation of the results of simulations. When controlled by the existing method, theoretical and experimental results were found to agree well with each other (see Fig. 9). Furthermore, it seems that the slight differences between the theoretical and experimental results occurred as a result of the fact that the spectrum at the control point could not be controlled exactly with the specified reference spectrum.

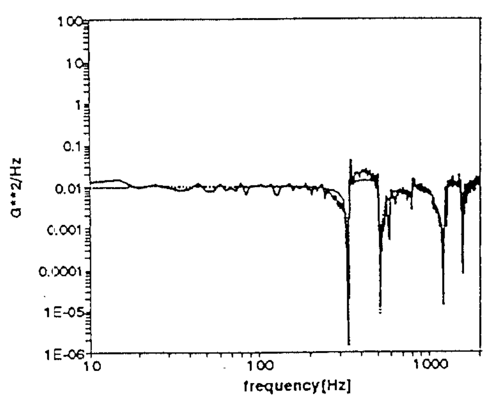
Figure 10 shows the results when the system



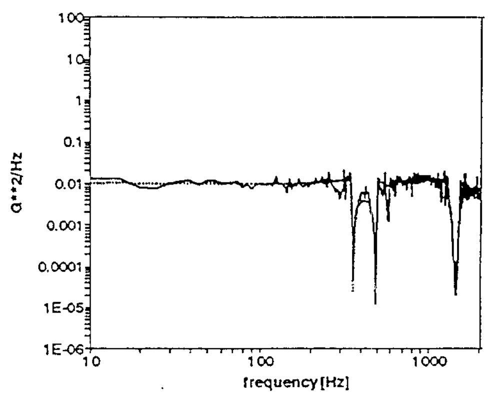
(a) Controlled PSD at control point, C_1



(c) Controlled PSD at specimen attachment point # 2, S_2



(b) Controlled PSD at specimen attachment point # 1, S_1



(d) Controlled PSD at specimen attachment point # 3, S_3

Fig. 10 Comparison of theoretical and experimental results of the proposed method (Dot : theoretical, Solid : experimental) is controlled using optimal reference spectrum as reference spectrum. In this case, experimental results are also well accordance with theoretical one.

4. Conclusion

This paper proposed a method for the determination of optimal reference spectrum in order to improve the existing methods, which control only the control accelerometer attachment point so that the spectra at the specimen attachment points are much different from the specified reference spectrum. The usefulness of the proposed method was also verified by conducting random vibration test of fixture.

Using the proposed method of the determination of the reference spectrum makes it possible to maximally reduce the overtests due to the dynamic characteristics and interactions of shaker, fixture and specimen and also to easily predict the response acceleration spectra at the specimen attachment points before conducting actual vibration test. In this way, because more exact vibration test can be conducted, so that clarifing and analyzing the causes of the problem such as malfunctions of specimen fracture or performence deterioration, etc., can be exactly done during or after vibration test the results can be effectively used considered in design modification process.

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